RECOVERY OF SOME IMPORTANT PARTIAL DIFFERENTIAL EQUATIONS FROM THE OYIBO GRAND UNIFICATION THEOREM†

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Abstract

The Oyibo grand unification theorem (GUT) has been proposed as a mathematical basis for a grand unification theory also known as the theory of everything. It is still in the exploration phase to recover already known results within the four main forces in nature before using it to explore new predictions. There is consensus that partial differential equations (PDEs) describe many of the fundamental processes of the physical world. Therefore, in this study we have formulated a generic PDE from the Oyibo GUT. From this generic PDE, we recover some PDEs important to mathematical physics. We are then led to conclude that the Oyibo GUT is a potential candidate for the grand unification theory as in our previous studies.

KEY WORD: Grand unification theorem, Unified force field, Wave component, Generic, Partial differential equations,

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1. INTRODUCTION

The Oyibo grand unified theorem (GUT) is (Oyibo 2003; Oyibo 2004) 'a physically sound or credible set of mathematical equations from which to determine or formulate the Grand Unified Force Field Theory.' Put more lucidly, the Oyibo GUT is a theorem comprising of a set of equations from which we can construct the mathematical formulations of all known and unknown forces in the universe and the resulting formation in its totality is the God Almighty Grand Unification Theory (GAGUT) also known as the theory of everything. Unfortunately, the Oyibo work has been criticized more for some of Oyibo ambitiously speculated applications of the GAGUT rather than the GUT which is a sound mathematical foundation from which we can construct physical formulations involving all the aforementioned forces in principle (Animalu, 2007a). Thus our interest in the Oyibo GUT is to explore its mathematical beauty as a theorem from which we can construct the mathematical formulations to account for all forces (Akpojotor and Echenim, 2010). We may not be able to immediately recover some of Oyibo's speculations especially those that are spiritual. However, it is possible we recover already known results involving the four main forces. Thereafter, we can begin to seek the formations of new predictions from the GUT. This may seems a very daunting task as the recovery of already known results alone may requires expertise adotion of the Oyibo GUT in the various field of physics. However, the possibility of having a unified force field theory from the Oyino GUT at the end is motivational enough to actuate such a study.

In previous studies (Echenim and Akpojotor, 2009; Akpojotor and Echenim, 2010), we have explored the hierarchy of the invariant solution for the case when n = 0 and try to recover some known standard equations of physics. In particular, we were able to recover results in geometric optics for both the Snell's law and the law of refraction from the generic solution for the motion of wave, η_0. As a continuation of our line of thinking of recovering previous results from the Oyibo GUT, the purpose of the study here is to construct a generic partial differential equation (PDE) from the Oyibo GUT in section 2 and then recover some special PDEs that have important applications in physics in section 3. This will be followed by a conclusion in section 4.

2. CONSTRUCTION OF THE GENERIC PDE FOR WAVE MOTION

The Partial differential equation is a mathematical equation that involves two or more independent variables, a dependent variable (and therefore the unknown function) and the partial derivatives of the unknown function with respect to the independent variables. It is now a consensus that the fundamental processes of the natural world is based to a large extent on partial differential equations (Farlow, 1993; Strauss, 2008;

Tikhonov and Samarskii, 2011). The implication is that if we can construct a generic PDE from the Oyibo GUT from which we can then recover the already known PDEs, then we would have also succeeded in showing that the Oyibo GUT can possibly be used to formulate the fundamental processes of the natural world.

The Oyibo GUT states that (Oyibo 2003):

The Grand Unified Force Field Theory can be derived from the generic universal set of conservation equations.

The basic implication of the above theorem is that all equations in physics will be subset of the conservation equations. Thus for this current study, the corollary is:

Corollary: *The partial differential equations important in physics form a subset of the generic universal conservation equations.*

Proof:

The Oyibo GUT set of conservative equations are

$$(G_{0n})_t + (G_{1n})_x + (G_{2n})_y + (G_{3n})_z = 0, \tag{1}$$

and its generic solution is

$$\eta_n = g_{n0}(ct)^{n+1} + g_{n1}x^{n+1} + g_{n2}y^{n+1} + g_{n3}z^{n+1} \tag{2}$$

where η_n which is a function of space-time coordinates (t, x, y, z) and the metric parameters ($g_{00}, g_{11}, g_{22}, g_{33}$) as well as n = 0, 1, 2, 3, 4. It has been demonstrated that it has the following hierarchy of invariant solution (Animalu,2007a; Akpojotor and Echenum, 2010):

$$\eta_0 = g_{00}ct + g_{10}x + g_{20}y + g_{30}z \tag{3a}$$

$$\eta_1 = g_{01}(ct)^2 + g_{11}x^2 + g_{21}y^2 + g_{31}z^2 \tag{3b}$$

$$\eta_2 = g_{02}(ct^3) + g_{12}x^3 + g_{22}y^3 + g_{32}z^3 \tag{3c}$$

$$\eta_3 = g_{03}(ct)^4 + g_{13}x^4 + g_{23}y^4 + g_{33}z^4 \tag{3d}$$

The word ‘generic’ here means that the specific nature of the solution is determined by the initial/boundary conditions and is a standard mathematical approach in theoretical physics. For example, solving the second order differential equation will yield the same general solution with two constants of intergration which can only be obtained from the initial/boundary conditions to obtain the particular solution. Similarly, solving a partial differential equation with m independent variable using the method of separation of variable will lead to m second order differential equations with *m – 1* separation constants. Now the separation constants to be introduced for a physical system depends on its initial/boundary conditions. This can quickly be illustrtated with the well established application of the time independent Schrondinger equation to the hydrogen atom. In speherical polar coordinate, it can be separated into the radial and angular parts. To fulfil the mathematical demand that both parts are equal, we often introduce a separation constant which naturally become the orbital magnetic quantum number.

We have emphasized on the 'generic solution' to drive the point home that it is a standard mathematical approach so that if we have to recover physics from the Oyibo GUT, then we must learn to use the initial/boundary conditions of the physical system to recoved it from the the Oyibo GUT. Consequently, we will recover some of the special PDEs important in physics from the generic PDE to be constructed from the Oyibo by using the appropriate initial/boundary conditions as well as the physical systems specifics.

Now in the above hierarchy in Eqs.(3a) – (3d), the case n = 0 represents the wave motion so that $F(\eta_0)$ is the wave component of Eq.(1). The $F(\eta_0)$ emerges as part of the unified field by integrating the diagonal components of Eq.(1) over appropriate elemental areas (Oyibo 2003):

$$G = \int G_{11} dzdy = \int G_{22} dxdz = \int G_{33} dxdy = F(\eta_0) + f_{1n} t^n F_1(\eta_n) + F_1(\eta_n) \quad (4a)$$

where

$$F(\eta_0) = F_G(\eta_0) + F_{EM}(\eta_0) + F_{SF}(\eta_0) + F_{WF}(\eta_0) + F_{OF}(\eta_0)\,, \qquad (4b)$$

and
$$F_1(\eta_n) = F_{1G}(\eta_n) + F_{1EM}(\eta_n) + F_{1SF}(\eta_n) + F_{1WF}(\eta_n) + F_{1OF}(\eta_n) \qquad (4c)$$

while f_{1n} is a constant, $F_1(\eta_n)$ is the generalized classical Newtonian or Einsteinian component of the force field, F_G is a gravitational force field, F_{EM} is an electromagnetic force field, F_{SF} is a strong field, F_{WE} is weak force field, F_{OF} is other field and $t^n F_1(\eta_n)$ some new component of force field.

It is easy to see that taking into account Eq.(3a), the wave component of the force field $F(\eta_0)$ is a function of an explicit function of the space-time coordinates. Therefore, applying the technique of partial differentiation, the total differentiation without any approximation is (Riley et al., 2006; Polyanin and Nazaikinskii, 2016)

$$\partial F(\eta_0) = \frac{\partial F(\eta_0)}{\partial t}dt + \frac{\partial F(\eta_0)}{\partial x}dx + \frac{\partial F(\eta_0)}{\partial y}dy + \frac{\partial F(\eta_0)}{\partial z}dz \qquad (5)$$

It is straightforward to show from Eq.(3a) that,

$$dt = \frac{1}{g_{00}c}d\eta_0,\ dx = \frac{1}{g_{10}}d\eta_0,\ dy = \frac{1}{g_{20}}d\eta_0 \text{ and } dz = \frac{1}{g_{30}}d\eta_0. \qquad (6)$$

Taking into account Eq.(6), then Eq.(5) can be re-expressed as

$$\partial F(\eta_0) = \frac{1}{g_{00}c}\frac{\partial F(\eta_0)}{\partial t}d\eta_0 + \frac{1}{g_{10}}\frac{\partial F(\eta_0)}{\partial x}d\eta_0 + \frac{1}{g_{20}}\frac{\partial F(\eta_0)}{\partial y}d\eta_0 + \frac{1}{g_{30}}\frac{\partial F(\eta_0)}{\partial z}d\eta_0$$

$$= \left(\frac{1}{g_{00}c}\frac{\partial F(\eta_0)}{\partial t} + \frac{1}{g_{m0}}\frac{\partial F(\eta_0)}{\partial r}\right)\partial\eta_0$$

Hence,

$$\frac{\partial F(\eta_0)}{\partial\eta_0} = \left(\frac{1}{g_{00}c}\frac{\partial F(\eta_0)}{\partial t} + \frac{1}{g_{m0}}\frac{\partial F(\eta_0)}{\partial r}\right) \qquad (7)$$

where $r = x, y, z$ and $m = 1, 2, 3$.

Partially differentiating Eq.(7) a second time will yield

$$\frac{\partial^2 F(\eta_0)}{\partial\eta_0{}^2} = \frac{1}{g_{00}c}\frac{\partial^2 F(\eta_0)}{dt^2} + \frac{1}{g_{m0}}\nabla^2 F(\eta_0) \qquad (8)$$

where the Laplacian operator, $\nabla^2 = \frac{\partial^2}{dr^2}$.

Eq.(8) is the generic PDE of the wave motion of the Oyibo GUT.

3.0 RECOVERY OF SOME PDES IMPORTANT IN PHYSICS

The most important PDEs in physics are second order and these include the wave equation, heat (also known as diffusion) equation, Laplace's equation and Poisson's equation (Honerkamp and Hartmann, 1993; Shearer and Levy, 2015).

3.1 The wave Equation

The wave equation describes the physical processes of wave propagation and it is a second order PDE. To obtain the wave equation from Eq.(8), lets consider that Eq.(8) describe a flexible elastic string of length *l* stretched between two fixed end points at x = 0 and x = l that is set vibrating by displacing its midpoint a distance *l/2* from its rest position and releasing it with zero initial velocity as shown in Figure 1. The oscillation from the vibration will be only in the x direction at time t > 0. Therefore, Eq.(8) will be describing a one-dimensional (1D) process, that is, $F(\eta_0) = F(x,t)$. Further, the boundary condition that the end points remain fixed means the vibration terminates at these ends points so that

$$\frac{\partial^2 F(\eta_0)}{\partial \eta_0^{\ 2}} = 0 = \frac{1}{g_{00}c}\frac{\partial^2 F(\eta_0)}{dt^2} + \frac{1}{g_{10}}\nabla^2 F(\eta_0)\ . \tag{9}$$

Thus

$$\nabla^2 F(\eta_0) = \frac{1}{c^2}\frac{\partial^2 F(\eta_0)}{dt^2} \tag{10}$$

where $\frac{g_{10}}{g_{00}} = \frac{d\eta_0}{dx}\frac{dt}{d\eta_0} = \frac{dt}{dx} = \frac{1}{c}$ from Eq.(6), $\nabla^2 = \frac{\partial^2}{\partial x^2}$ as $\eta_0(t,x) = g_{00}t + g_{10}x$ and the omission of the negative sign is not an artefact but is assumed to have been absorbed in the metric parameters.

Eq.(10) is in the form of the 1D wave equation whose general 3D form is (Strauss, 2008; Kirkwood, 2012)

$$\nabla^2 U(r,t) = \frac{1}{c^2}\frac{\partial^2 U(r,t)}{\partial t^2}\ . \tag{11}$$

where c a real constant that can be interpreted as the corresponding wave speed. The emergence of this c naturally in our formation is a boost to our study. Thus we can re-express Eq.(10) in 3D taking into account that $F(\eta_0)$ is a function of an explicit function of the space-time coordinates, that is, $F(\eta_0) = F(r,t)$, as

$$\nabla^2 F(r,t) = \frac{1}{c^2}\frac{\partial^2 F(r,t)}{dt^2} \tag{12}$$

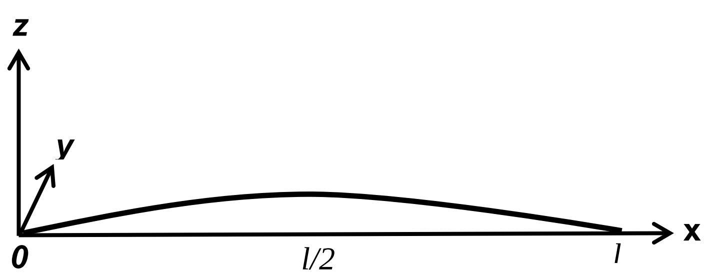

Figure 1: A flexible elastic string of length *l* stretched between two fixed end points at *x = 0* and *x = l* that is set vibrating by displacing its midpoint a distance *l/2* from its rest position and releasing it with zero initial velocity.

3.2 The Heat Equation

The heat equation is used to describe the process of heat distribution or variation in temperature in a given region over time with the possibility of linking the volume of the region of a manifold with its area. To recover the heat equation from Eq.(8), lets consider an infinitesimal volume element in a Cartesian coordinate system so that the dimensions of the infinitesimal volume element are *dx*, *dy* and *dz* in the respective direction as depicted in Figure 2. It is easy to envisage that the heat entering into the volume element from three different (area) faces of the volume element is leaving from the opposite (area) face of the control element after heat generation inside the element. The implication is that the difference between the rate of change of the energy content of the element and the rate of heat generation inside the element will be equal to the difference between the rate of heat conduction at *x*, *y* and *z* and the rate of heat conduction at *x* + *dx*, y + *dy* and z + *dz*. Thus there is conservation of energy so that

$$\frac{\partial^2 F(\eta_0)}{\partial \eta_0{}^2} = 0 = \frac{1}{g_{00}c}\frac{\partial^2 F(\eta_0)}{dt^2} + \frac{1}{g_{m0}}\nabla^2 F(\eta_0) \tag{13}$$

Taking the direct integration of Eq.(13) and assuming the integration constant can be absorbed into metric parameters, we obtain

$$\frac{1}{g_{00}c}\frac{\partial}{\partial t}\left(\frac{\partial F(\eta_0)}{dt}\right) = \frac{1}{g_{m0}}\nabla^2 F(\eta_0)$$
$$\left(\frac{\partial F(\eta_0)}{dt}\right) = \frac{g_{00}ct}{g_{m0}}\nabla^2 F(\eta_0) \tag{14a}$$

which can be recast as

$$\left(\frac{\partial F(\eta_0)}{dt}\right) = \kappa\nabla^2 F(\eta_0) \tag{14b}$$

where $\kappa = \frac{g_{00}ct}{g_{m0}}$ is a real constant whose deimensional unit is *length*^{2} *x time*$^{-1}$ in agreement with the diffusion equation (Selvadurai, 2000; Shearer and Levy, 2015) wherein it is refered to as difussivity. Thus the natural emergence of the κ with the appropriate unit again is a boost to our formulation here. For as pointed in Riley et al. (2006), the physical constants that will make up κ in a particular case depend on the nature of the process and material being described. These constants and the material specifics are assumed to be embedded in the metric parameters here. It follows then that if we take into account that $F(\eta_0)$ is a function of an explicit function of the space-time coordinates, that is, $F(\eta_0) = F(r,t)$, then Eq.(14b) can ne recast in Cartesian coordinates as

$$\left(\frac{\partial F(r,t)}{dt}\right) = \kappa \nabla^2 F(r,t) \,. \tag{15}$$

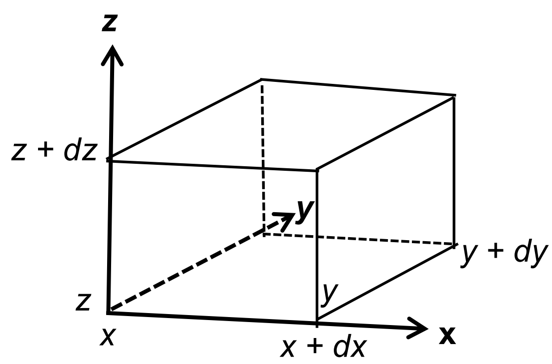

Figure 2: Heat distribution with time in an infinitesimal volume element in a Cartesian coordinate system so that the dimensions of the infinitesimal volume element are *dx*, *dy* and *dz* in the respective direction

3.3 The Laplace's Equation

The Laplace's equation describes the distribution of a field over a volume or an area subject to certain boundary conditions and long after the source has elapsed. This means the physical systems that (1) are steady states wherein the time is taken to be zero so that the generic solution will depends only on the space coordinates (2) the source of the distribution is considered zero after it has elapsed. Thus the Oyibo generic PDE in Eq.(8) becomes

$$\frac{\partial^2 F(\eta_0)}{\partial \eta_0{}^2} = 0 = \frac{1}{g_{m0}} \nabla^2 F(\eta_0) \,. \tag{16}$$

To illustrate the Laplace's equation using Eq.(16), lets consider a two infinite metal plates that are grounded and lie parallel to the y-*z* plane with one located at $x = 0$ and the other located at $x = x_1$, while the back side located at $y = 0$ is closed off with an infinite metal strip insulated from the two parallel planes as shown in Figure 3. This is a 2-dimensional problem as the field do not depends explicitly on z. Therefore Eq.(16) reduces to

$$\frac{\partial^2 F(\eta_0)}{\partial x^2} + \frac{\partial^2 F(\eta_0)}{\partial y^2} = 0 \tag{17}$$

where we have taken the metric paramters to be unity: $g_{10} = g_{20} = 1$.

Thus making appropriate case for $g_{m0} = 1$ in Eq.(16) and taking into account that $F(\eta_0)$ is a function of an explicit function of the space coordinates only, that is, $F(\eta_0) = F(r)$, Eq.(16) can be expressed as the Laplace's equation in 3D:

$$\nabla^2 F(r) = 0 \qquad (18)$$

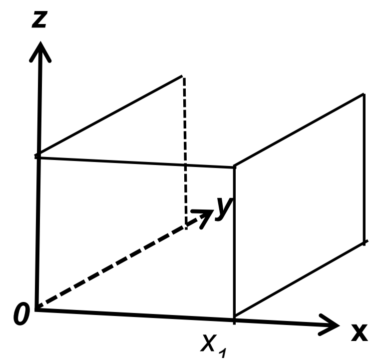

Figure 3: Two infinite metal plates that are grounded and lie parallel to the y-z plane with one located at $x = 0$ and the other located at $x = x_1$, while the back side located at $y = 0$ is closed off with an infinite metal strip insulated from the two parallel planes

3.4 The Poisson's Equation

The Poisson's equation is like the Laplace's equation except that its source has not elapsed (Lamb Jr, 1995, Shearer and Levy, 2015). The implication is that it behaves as the inhomogeneous Laplace's equation (Olver, 2014):

$$\frac{\partial^2 F(r)}{\partial \eta_0^{\ 2}} = \nabla^2 F(r) \qquad (19)$$

where $g_{m0} = 1$ and $\eta_0 = \eta_0(r)$.

Now what constitute the inhomogeneous part of Eq.(19) which is often refered to as the source density depends on the nature of the problem. The general feature of this inhomogeneous part is that it is an explicit function of only the space coordinate so that

$$\frac{\partial^2 F(r)}{\partial \eta_0^{\ 2}} \equiv F(x, y, z) = \nabla^2 F(r). \qquad (20)$$

For gravitation, it has been pointed out that the Newton's universal gravitational force field satisfies the Poisson equation (Oyibo, 2003):

$$\nabla^2 F(r) = 4\pi G p \qquad (21)$$

where ρ is the density of an attracting massive object and G is the gravitational constant.

For electrostatic, it is a text book knowledge that the inclusion of the inhomogeneous part of Eq.(20) made up of the electric charge ρ and the permittivity of vacuum, ε_0, will yield the Poisson's equations for electrostatic:

$$\nabla^2 F(r) = \frac{-\rho}{\varepsilon_0}. \qquad (22)$$

4. CONCLUSION

We have been able to proved the corollary that *the partial differential equations important in physics form a subset of the generic universal conservation equations*. In particular, we have been able to recover the wave equation, heat equation, Laplace's equation and Poisson's equation which are very important PDEs from the wave component of the Oyibo GUT. In principle, it follows then that all the many and varied applications of these PDEs in physics currently (Shearer and Levy, 2015; Polyanin and Nazaikinskii, 2016) as well as the ones that are likely to be discovered in future, can also be obtained from the Oyibo GUT. Thus we are reaching the same conclusion in this current study as in previous studies (Animalu, 2007a; Animalu, 2007b; Echenim and Akpojotor, 2009; Akpojotor and Echenim, 2010) that the Oyibo GUT has the sound mathematical foundation to be a potential candidate of a grand unification theory.

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