GENO-SUPERCONDUCTIVITY OF QUASICRYSTALS†

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Abstract

An O(4,2)xSU(3)xU(1) lattice gauge group that decomposes to SO(10) approach to strong coupling ("holographic") superconductivity is developed under the name *geno-superconductivity* in order to incorporate geno-Bragg's law obeyed by quasicrystal structures as well as non-trivial conformal (including scale) invariance. As a progressive generalization of the *iso-superconductivity theory* based on non-local, non-Hamiltonian formulation of the standard (Bardeen, Cooper and Schrieffer(BCS)) model, geno-superconductivity theory is founded, in algebraic and geometric terms, on extensions of the underlying Lie-algebraic structure of the BCS model to include ellipsoidal and toroidal deformations of a spherical Fermi surface. Although the BCS model Hamiltonian can be diagonalized exactly in Wannier representation, in terms of 2^Nx2^N-matrix representation of creation and annihilation operators for any number (N) of electrons corresponding to a 2^N-dimensional representation of the spinor group SO(2N), for N= 2,3,4,5,… with SO(8) and SO(10) containing the group SO(5) that unites the superconducting phase in the BCS model and antiferromagnetic phase in the t-J Hubbard model, geno-Bragg's law demands that the observed diffraction pattern of quasicrystal structures with unexpected 10-fold point symmetry previously explained in terms of 5+5=10-fold point symmetry should be re-interpreted as a cube-hexagon 4+6=10-fold point-symmetry in favour of SO(10). As a result, we have constructed a geno-London equation in O(4,2)xSU(3)xU(1) lattice gauge that predicts the *emagnetodynamics* principle of operation of composite magnetic poles which has an unexpected feature of perpetually driving a motor with static magnetic field. The results are discussed.

Keywords: Standard BCS model, geno-Bragg's Law, quasi-crystals, London equation, holographic superconduct, antiferromagnetism, emagnetodynamics.

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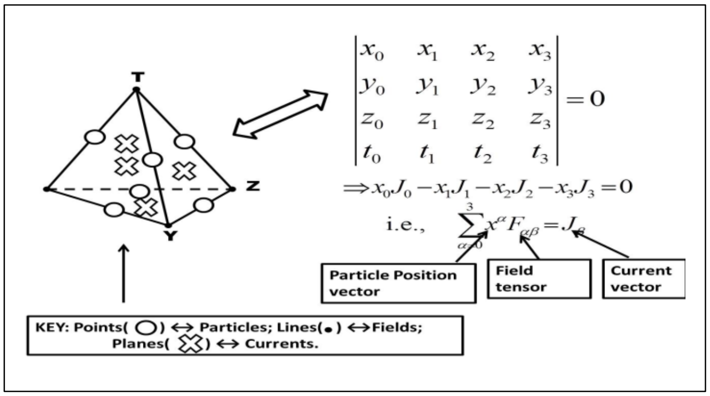
1. INTRODUCTION

Among the various competing models of strong coupling ("holographic") superconductivity (see, for example, the review by Abah *et al*(2012)[1]), a lattice gauge O(4,2)xSU(3)xU(1) group that decomposes to SO(10) approach based on space-time geometry of a torus and conformal (including scale) invariant string theory without divergences(A.O.E. Animalu and C.N. Animalu(2010)[2]), is very promising for incorporating geno-Bragg's law obeyed by quasicrystal structures with unexpected 10-fold point symmetry(Schchtman and co-workers(1984)[3]) in a progressive generalization, under the name *geno-superconductivity*, of the *iso-superconductivity theory* (Animalu(1994)[4] and Animalu and Santilli(1995)[5]). In algebraic and geometric terms, such a generalization from iso- to geno-superconductivity may be looked upon as an extension of the underlying Lie-algebraic structure of the standard (Bardeen, Cooper and Schrieffer(1957)[6] (BCS)) model based on pairing of electrons across a spherical Fermi surface to include ellipsoidal and toroidal deformations of the Fermi sphere. Algebraically, because the BCS model Hamiltonian can be diagonalized exactly(C.N. Animalu(1992)[7]) in Wannier representation, in terms of $2^N x 2^N$-matrix representation of creation and annihilation operators for any number (N) of electrons corresponding to a 2^N-dimensional representation of the spinor group SO(2N), for N= 2,3,4,5,… with SO(8) and SO(10) containing the group SO(5) that unites the superconducting phase in the BCS model and antiferromagnetic phase (Zhang(1997)[8]) in the Zhang-Rice(1988)[9] t-J Hubbard model, the constraint by geno-Bragg's law (Animalu(2013)[10]) favours SO(10). In addition, SO(10) provides a realization of Varma's(1989)[11] current loop model of high-Tc superconductivity in which electric current loops form spontaneously, going from copper to oxygen atoms and back to copper in the cuprates, and the coexistence of superconductivity in the BCS model with antiferromagnetism in the variant derived by Akpojotor's[12](2008) from superexchange pairing mechanism in the CuO_2 planes of the cuprates with a successful application by Animalu *et al* (2009) [13] to the high-T_c superconductivity of the iron pnictides observed by Hunte et al(2008) [14]. However, the discoveries of the existence of two gaps and 3-dimensional(3D) composite-crystal nature of superconductivity in the pnictides have stimulated interest in various models: including the two-band electron pairing extension of the one band BCS model to two bands using the Green's function method(Asomba (1995)[15]) as well as exploration of correspondences between anti-DeSitter/Conformal Field Theories (AdS/CFT) of black holes in quantized space-time lattice and strongly coupled ("holographic") superconductors(see, for example, Hartnoll, et al (2008)[16]).

It is with the above motivation that we shall, in this paper, develop geno-superconductivity theory firstly (in Sec. 2) from an extension of the standard BCS model and secondly (in Sec.3) as a generalization of iso-superconductivity that incorporate geno-Bragg's law for quasicrystal structures developed by Animalu(2013)[10]) in the framework of the correspondence principle:

points⇔ particles, lines ⇔ fields, planes ⇔ currents,

between points, lines and planes in algebraic projective geometry of 3D space and particles, fields and (conserved) currents of Maxwell-type gauge field theories (see, Fig.1)



in the following explicit and extended form:

Geometry: No. of points + lines + planes = 4+6+4 =14 | **Geno-Bragg's Law** | **O(4,2)xSU(3)xU(1) Lattice gauge theory**

$$\left\{\begin{array}{l}(\mathbf{k}\Leftrightarrow\mathbf{p},\ \mathbf{g}\Leftrightarrow\mathbf{J}\equiv e\mathbf{v}\text{ or }e\gamma)\\(\mathbf{k}\Leftrightarrow\mathbf{J},\ \mathbf{g}\Leftrightarrow\mathbf{B})\end{array}\right.\Rightarrow\left\{\begin{array}{l}\mathbf{g}^2\pm(\mathbf{g.k}+\mathbf{k.g})\\(\mathbf{k}^2\pm(\mathbf{k.g}+\mathbf{g.k})\end{array}\right\}\Leftrightarrow\left\{\begin{array}{l}\mathbf{p.p}\pm M^{-1}(\mathbf{p.J}+\mathbf{J.p})\\\mathbf{J.J}\pm(\mathbf{J.B}+\mathbf{B.J})\end{array}\right\}\quad(1.1)$$

Here $\mathbf{k}$ and $\mathbf{g}$ are the conventional wave vector and reciprocal lattice vector, $\mathbf{p},\mathbf{J},\mathbf{B}$ are the momentum, current and magnetic field three-vectors, $e\mathbf{v}$ or $e\boldsymbol{\gamma}$ is the electric current associated with particle velocity(**v**) or Dirac $\boldsymbol{\gamma}$ –matrix, e is the electric charge, and M a characteristic mass in units such that $\hbar = c = 1.$ Whereas the first part of the correspondence in Eq.(1.1) will be relied upon in Sec.3 of this paper for incorporating the generalization of the conventional Bragg's law of reflection of light and matter waves to geno-Bragg's law $\mathbf{g}^2 = \pm(\mathbf{g.k}+\mathbf{k.g})$ for quasi-crystals structures, in quantum theory, the term $\mathbf{p.p} \pm M^{-1}(\mathbf{p.\gamma}+\mathbf{\gamma.p})$ given by $\mathbf{J} \Leftrightarrow \boldsymbol{\gamma}$ determines the most general parity-conserving (vector and "convective") current in the O(4,2) algebra of Dirac $\boldsymbol{\gamma}$-matrices and its O(4,2)xSU(3)xU(1) lattice gauge determines mass ratios of SU(3) multiplets of strongly interacting elementary particle masses[17]. The analogous current and magnetic field part $\mathbf{J.J} \pm (\mathbf{J.B}+\mathbf{B.J})$ will be used in Sec. 4 to generalize the conventional London's (1961)[18] equation for the Meissner effect to a geno-

London equation, that will predicts the *emagnetodynamics* principle (Animalu[2013](19) of operation of composite magnetic poles machine invented by Dr. Ezekiel Izuogu[20]) which has an unexpected feature of perpetually driving a motor with static magnetic field. The results will be discussed and conclusions drawn in Sec.5

2. GENO-BCS MODEL OF SUPERCONDUCTIVITY

It is well-known that the quasiparticle energy of the BCS model of superconductivity has the following form:

$$E_k^2 = \varepsilon_k^2 + \Delta_k^2 \qquad (2.1)$$

relating the free-electron energy $\varepsilon_k \equiv (\hbar^2/2m)(k_x^2 + k_y^2 + k_y^2)$ measured relative to the Fermi energy, and the energy gap (Δ_k). Note that in Eq.(2.1) ε_k is the radius of a spherical (Fermi) surface in wave vector, $\mathbf{k} = (k_x, k_y, k_z)$-space but the usual parameteric representation (see, e.g., p.442 of Animalu(1977)[21])

$$\varepsilon_k / E_k = v_k^2 - u_k^2,\ \Delta_k / E_k = -2u_k v_k,\ u_k^2 + v_k^2 = 1, \qquad (2.2a)$$

defines a (deformation) ratio,

$$x \equiv \varepsilon_k / \Delta_k = (u_k^2 - v_k^2)/2u_k v_k = du_k / dv_k, \text{ i.e. } (-u_k^2 + v_k^2) + 2xu_k v_k = 0. (2.2b)$$

Physically, v_k^2 represents the momentum distribution of electrons in the BCS ground state while the amplitude $u_k v_k$ represents the overlap of the Cooper pair wavefunction and penetrates the smear in v_k^2 at k_F (the Fermi wavenumber) over an interval $1/\zeta_0 \sim \Delta_0 / hv_F$ where ζ_0 is of order $10^{-4} cm$, v_F being the Fermi velocity (see, Fig. 2)

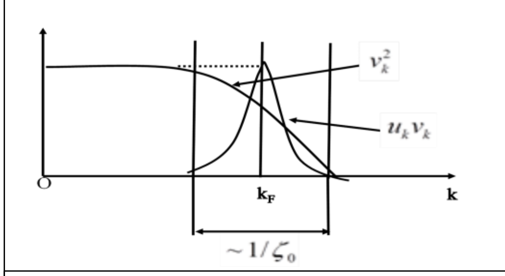


Fig.2: k-dependence of $u_k v_k$ and v_k^2.

An apparent generalization arises by relating Eqs.(2.1) and (2.2b) formally as a deformation in $(u_k, v_k)-$space as follows:

$$1 = v_k^2 + u_k^2 \equiv \begin{pmatrix} u_k & v_k \end{pmatrix} \begin{bmatrix} 1 & -x \\ x & 1 \end{bmatrix} \begin{pmatrix} u_k \\ v_k \end{pmatrix}$$

$$\to \begin{pmatrix} u_k & v_k \end{pmatrix} \begin{bmatrix} -1 & x \\ x & 1 \end{bmatrix} \begin{pmatrix} u_k \\ v_k \end{pmatrix} \equiv (-u_k^2 + v_k^2) + x(u_k v_k + v_x u_x) = 0. \quad (2.3)$$

which can be expressed as a (*left or right genotopy*) transformation of an underlying non-singular matrix(τ) as follows:

$$\tau DT \equiv \begin{pmatrix} -1 & x \\ x & 1 \end{pmatrix} \leftarrow \tau \equiv \begin{pmatrix} 1 & -x \\ x & 1 \end{pmatrix} \equiv \tau \to \begin{pmatrix} -1 & x \\ x & 1 \end{pmatrix} = TC\tau \qquad (2.4a)$$

Therefore, it leads to a readily verifiable *Lie-admissible* algebraic relation

$$TC\tau - \tau DT = 0 \;\; (C \neq D \neq I) \qquad (2.4b)$$

where, $C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv \sigma_1, T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \equiv i\sigma_2, D \equiv \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \equiv -\sigma_3.$

in terms of Pauli spin matrices, $(\sigma_1, \sigma_2, \sigma_3)$ However, in general, to ensure that the "geno-units" (C and D) are invertible and positive definite one must define for each quantity, $Q \equiv Q(x)$ say, *two non-unitary* transformations,

$$CC^+ \neq I,\; DD^+ \neq I,\; CD^+ \neq I,\; Q \to \hat{Q}^> = CQD^+, Q \to \hat{Q}^{>+} = DQC^+ \;(2.5a)$$

so that if τ is re-interpreted as a Hamiltonian operator, $\hat{H} \neq \hat{H}^+$, then as a generalization of (2.4b), Q obeys a forward time-evolution law of Santilli's (2006)[22] Lie-admissible form:

$$\hat{i}^> > \hat{\hat{d}}^> \hat{Q}^> / \hat{\hat{d}}^> \hat{t}^> \; = \; \hat{Q}^> > \hat{C}^> > \hat{H} - \hat{H} > \hat{D}^> > \hat{Q}^> . \qquad (2.5b)$$

and similarly for the backward time-evolution. The form in Eq.(2.5b) exhibits the essential time-irreversibility of the geno-BCS model envisaged in Varma's current loop model of high-T_c superconductivity[11] in which quantum-mechanical fluctuations are time-dependent fluctuations of the current loops.

To study the current-loop representation, we separate Eq.(2.2b), i.e.,

$$du_k / dv_k = (v_k^2 - u_k^2)/2u_k v_k, \qquad (2.6a)$$

in the form (involving a Riccati's equation):

$$\frac{du_k}{d\beta} = \kappa(-u_k^2 + v_k^2) \;\; \text{(Riccati's equation)}; \frac{dv_k}{d\beta} = 2\kappa u_k v_k \;\; (2.6b)$$

Equivalently, we can rewrite Eq.(2.5a) in the form

$$\frac{du_k}{dv_k} = \frac{-u_k^2 + v_k^2}{2u_k v_k} = \tfrac{1}{2}(-u_k / v_k + v_k / u_k), \text{ or } \; du_k^2 / dv_k = -u_k^2 / v_k + v_k \;\; (2.7a)$$

which has an exact solution in the form (cf Eqs.(1.3.10) and (1.3.11) of J. Guckenhimer and P. Holmes (1983)[23])

$$u_k^2 = v_k^2/3 + u^3/3v_k\,, \quad \text{i.e. } v_k^3 - 3u_k^2 v_k + u^3 = 0 \qquad (2.7b)$$

where $u^3/3$ is an arbitrary constant. Consequently, by putting $u_k^2 = \omega u$ and dropping the suffix in both u_k and v_k we obtain the relation:

$$u^3 + v^3 = 3uv\omega \qquad (2.8)$$

which describes a curve in $u-v$ space known as the *folium* of Descartes[24] – a current loop that may be correlated with observed light "caustics"[25] as in Fig.3.

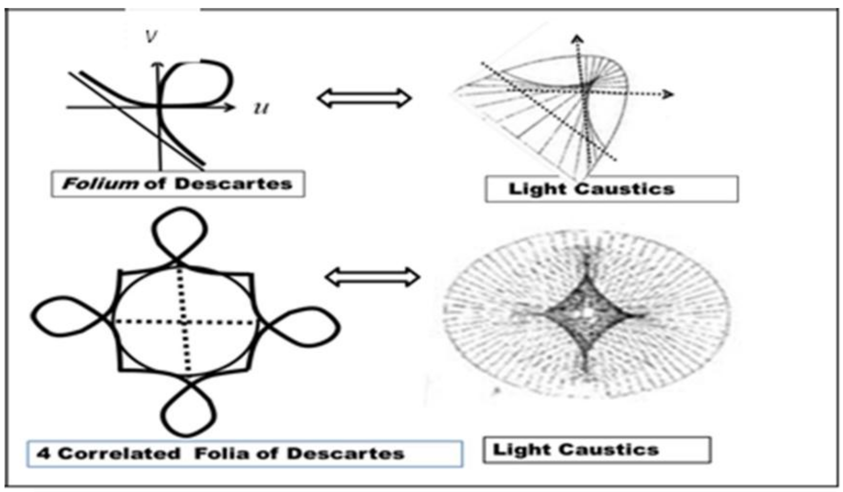


Fig.3: Correlated Folium of Descartes (current loops) and Light "Caustics"

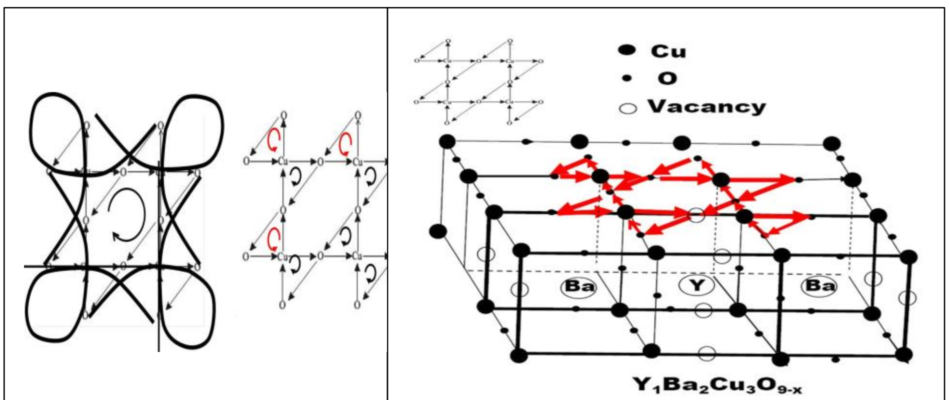


Fig.4(Left) Correlation of network of Folium of Descartes with Varma's model of superconductor & Akpojotor's superexchange pairing mechanism and (right) application to CuO_2 planes of a superconducting cuprate.

Such a correlation of folium of Descartes with a current loop, can be used to define a network of current loops in the CuO_2 planes of the high-T_c cuprate superconductor as in Fig. 4 in agreement with the current loop model of high-Tc superconductivity proposed by Chandra Varma(1989)[11] and reformulated as superexchange pairing mechanism by Akpojotor's(2008)[12]].

If we formally put $v_k = r/t$ in Eq.(2.6b), we find the "cubic catastrophe"[25]

$$r^3 + (ut)^3 = 3r(ut)^2, i.e., X + Yr + r^3 = 0, \tag{2.9a}$$

where $(X, Y) = ((ut)^3, -3(ut)^2)$, and with $r = r_o \equiv v_k / \omega$, in Eq.(2.6b) we find

$$u_k^2 = v_k^2/3 + u^3/3v_k \equiv \frac{(v_0 - \omega r)^2}{3} + \frac{u^3}{3(v_0 - \omega r)} \equiv \frac{\omega^2 (r - r_0)^2}{3} + \frac{u^3/3\omega}{(r - r_0)}. \tag{2.9b}$$

Since this is analogous to Schwarzschild "black hole" singularity(Frankel (1979)[26]) of Einstein's theory of general relativity, it is of interest to find a natural framework for the correspondence between AdS/CFT of "black holes" in quantized space-time lattice and strongly coupled ("holographic") superconductors[17]. We generalize the usual (static) de-Sitter space metric to a screw (dynamic) geno-de Sitter space metric by adding a *deformation* term $2\zeta drdt$ (ζ being an arbitrary parameter measure of the sine of the angle between $d\vec{r}$ and $d(\vec{c}t)$) to the conventional Schwarzschild line element of Einstein's general relativity theory in spherical polar coordinates (θ, φ, r, ct) (c=1):

$$(ds)^2 = r^2[(d\theta)^2 + \sin^2\theta(d\varphi)^2] + \frac{1}{(1 - \frac{2M}{r})}(dr)^2 - (1 - \tfrac{2M}{r})(dt)^2 + 2\zeta drdt \tag{2.10a}$$

In geometric terms, the envisaged progressive generalization from the conventional (static) to screw (dynamic) de-Sitter space as illustrated in Fig. 5 is of the form:

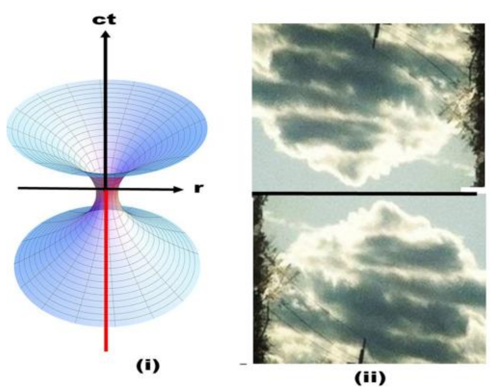


Fig.5: Representations of (i) conventional (static) de-Sitter space and (ii) its screw (dynamic) view as unusual cloud formation (sourced/adapted from America-on-line AOL Sept 2, 2013) defining a vortex as a current loop.

$$(ds)^2 = r^2[(d\theta)^2 + \sin^2\theta(d\varphi)^2] + (dt, dr)\begin{pmatrix} -K & \zeta \\ \zeta & K^{-1} \end{pmatrix}\begin{pmatrix} dt \\ dr \end{pmatrix} \qquad (2.10b)$$

where $(1 - \frac{2M}{r}) \equiv K$ leads to the "black hole" singularity in the limit $K \to 0$.

By transforming the last term in (2.10b) as follows:

$$(-K(dt)^2 + K^{-1}(dr)^2 + \zeta(drdt + dtdr) \equiv (dt, dr)\begin{pmatrix} -K & \zeta \\ \zeta & K^{-1} \end{pmatrix}\begin{pmatrix} dt \\ dr \end{pmatrix}.$$

$$\to (dt', dr')\begin{pmatrix} K & -\zeta \\ \zeta & K^{-1} \end{pmatrix}\begin{pmatrix} dt' \\ dr' \end{pmatrix} \equiv K(dt')^2 + K^{-1}(dr')^2 + \zeta(dt'dr' - dr'dt') \qquad (2.11)$$

then for K=1 the underlying metrics would be related by the *left* genotopy:

$$\eta \equiv \begin{pmatrix} -1 & \zeta \\ \zeta & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -\zeta \\ \zeta & 1 \end{pmatrix} = -TC\eta \qquad (2.12)$$

But if the last term is transformed as follows

$$-\zeta^{-1}(K(dt)^2 + K^{-1}(dr)^2) + 2drdt \equiv (dt, dr)\begin{pmatrix} -K\zeta^{-1} & 1 \\ 1 & K^{-1}\zeta^{-1} \end{pmatrix}\begin{pmatrix} dt \\ dr \end{pmatrix}$$

$$\to (dt', dr')\begin{pmatrix} K & -\zeta^{-1} \\ \zeta^{-1} & K^{-1} \end{pmatrix}\begin{pmatrix} dt' \\ dr' \end{pmatrix} \to (dt')^2 K + (dr')^2 / K - \zeta^{-1}(dr'dt' - dt'dr'). \qquad (2.13)$$

then for K=1, the underlying metrics would be related by the *right* genotopy

$$\eta \equiv \begin{pmatrix} -\zeta^{-1} & 1 \\ 1 & \zeta^{-1} \end{pmatrix} \to \begin{pmatrix} 1 & -\zeta^{-1} \\ \zeta^{-1} & 1 \end{pmatrix} = -\eta DT; \qquad (2.14)$$

Consequently, if $\zeta = 1$, we infer that $TC\eta - \eta DT = 0$ ($C \neq D \neq I$), is a Lie-admissible algebraic relation, like Eq.(2.4b), which is the result we are after.

Finally, a generalization of geno-BCS to two-band representation of electron pair states Hamiltonian, is provided by the model used to characterize the iron pnictide, $NdFeAsO_{0.9}F_{0.1}$, in the following linearized form(Igwe(2011)[27]:

$$H = \sum_{i=1,2;k} A_{ik}[d^+_{ik\uparrow}, d_{ik\downarrow}]\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} d_{ik\uparrow} \\ d^+_{ik\downarrow} \end{pmatrix} - \sum_{i=1,2;k} \Delta_{ik}[d^+_{ik\uparrow}, d_{ik\downarrow}]\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} d_{ik\uparrow} \\ d^+_{ik\downarrow} \end{pmatrix}$$

$$\equiv \sum_{i=1,2;k} [d^+_{ik\uparrow}, d_{ik\downarrow}]\begin{pmatrix} A_{ik} & -\Delta_{ik} \\ -\Delta_{ik} & -A_{ik} \end{pmatrix}\begin{pmatrix} d_{ik\uparrow} \\ d^+_{ik\downarrow} \end{pmatrix} \qquad (2.15a)$$

where $A_{ik} = \varepsilon_{ik} - \mu$, with ε_{ik} being the quasi-particle energy of the band electrons (i=1,2) and chemical potential μ, and the energy gaps,

$$\Delta_{1k} = \sum_k (V_{1kk'} S_{ik} + V_{12} S_{2k}),\ \Delta_{1k} = \sum_k (V_{1kk'} S_{ik} + V_{12} S_{2k}), \qquad (2.15b)$$

are given in terms of the intra-band and (inter-band), electron interaction $V_{1kk'}, V_{2kk'}\,(V_{12kk'})$ for first band electron pair with wave vector k, k' measured relative to the screened Coulomb potential as well as the pair correlations, $S_{ik} = \langle d_{ikS} d_{ikS} \rangle = \langle d^+_{ikS} d^+_{ikS} \rangle, i = 1{,}2$ arising from Gor'kov factorization. Consequently, by rewriting the pseudospin matrix in Eq.(2.15a) in the form:

$$\begin{pmatrix} A_{ik} & -\Delta_{ik} \\ -\Delta_{ik} & -A_{ik} \end{pmatrix} = -A_{ik} \begin{pmatrix} -1 & \xi \\ \xi & 1 \end{pmatrix} \qquad (2.16)$$

where $\xi \equiv \Delta_{ik} / A_{ik}$, we readily infer that it can be expressed as *left and right genotopy* of a matrix(τ) as follows

$$\tau DT \equiv \begin{pmatrix} -1 & \xi \\ \xi & 1 \end{pmatrix} \leftarrow \tau \equiv \begin{pmatrix} 1 & -\xi \\ \xi & 1 \end{pmatrix} \equiv \tau \rightarrow \begin{pmatrix} -1 & \xi \\ \xi & 1 \end{pmatrix} = TC\tau \qquad (2.17a)$$

From this, we again obtain a *Lie-admissible* algebraic relation, like Eq.(2.4b),

$$TC\tau - \tau DT = 0 \ (C \neq D \neq I) \qquad (2.17b)$$

On gathering results, the recurrence of this Lie-admissible algebraic relation, demands that we include the constraints from Dirac's dual (electric and magnetic) charge quantization rule [28] on various charge carriers involved in geno-BCS theory namely, leptons and baryons as Cooper pairs, as well as different types of gauge particles(photons, phonons, magnons, gravitons). For this purpose, we may start by extending the conventional Dirac's quantization rule $(eg = n\hbar c,\ n = \text{integer})$ to geno-Dirac quantization rule for electric and magnetic charges (e, g) when the speed of light(c) and Planck's constant$(\hbar)$ are dependent on a common local variable as follows:

$$(g^2 - e^2) + 2\frac{dg}{de} eg = 0. \text{ i.e. } \frac{d(n\hbar c)}{de} = -\frac{n\hbar c}{e} + e. \qquad (2.18a)$$

Like Eq.(2.6a) this has an exact solution

$$n\hbar c = e^2/3 + e_0^3/3e = 0. \text{i.e. } e^3 + e_0^3 = 3n\hbar ce. \qquad (2.18b)$$

$e_0^3/3$ being an arbitrary constant, from which it follows that (for $e_0 = 0$), $g = n\hbar c / e = e/3$ is a possible value of the magnetic charge which agrees with the suggestion by Recami and Mignani[29] to identify a magnetic monopole carrying magnetic charge (g) with a fractionally-charged quark carrying electric charge e/3, and with the standard model of SU(3), in which the quarks triplet (u, d, s) have fractional baryon number(B) and fractional electric charge (eQ_B) in unit of the proton electric charge (e=1),

$$B = \left(\frac{1}{3}\right)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, Q_B = \left(\frac{1}{3}\right)\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad (2.19a)$$

However, the corresponding lepton triplet $\ell = (\nu, e^-, \mu^-)$ have integral lepton number (L) and electric charges(eQ_L):

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, Q_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \qquad (2.19b)$$

and by observing that,

$$(B + Q_B) = (L + Q_L) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \equiv F + Q_F = P \qquad (2.19c)$$

is idempotent (i.e., $P^2 = P$), it follows that the quarks can be obtained from the leptons by (non-unitary) shifting of 2/3 of the lepton number to the leptonic charge, i.e., $B \equiv (L - \frac{2}{3}L), Q_B \equiv (Q_L + \frac{2}{3}L)$, leading to the complementary duality of baryons and leptons[30], characterized by a non-unitary transformation

$$U^+ Q_B U \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\sqrt{3} \\ 0 & +\sqrt{3} & 0 \end{pmatrix}\begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sqrt{3} \\ 0 & -\sqrt{3} & 0 \end{pmatrix}.$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \equiv Q_L \qquad (2.20)$$

However, to make the non-unitary transformation non-singular, we may replace the matrices, U and U^+, by

$$W \equiv I + U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \sqrt{3} \\ 0 & -\sqrt{3} & 1 \end{pmatrix}, \ Z = I - U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\sqrt{3} \\ 0 & +\sqrt{3} & 1 \end{pmatrix}, \ (3.14)$$

This demands, for mathematical consistency, the use of Santilli's Lie-admissible generalization indicated in Eqs.(2.5a&b) of the conventional Lie-algebraic structure of quantum mechanics, with the consequence that magnetic monopoles (current loops) can be used to replace the paired electrons in fractional quantum Hall effect in geno-London theory, to which we shall return in Sec.4.

3. GENERALIZATION OF ISO- TO GENO-SUPERCONDUCTIVITY

We turn next to a formal construction of geno-superconductivity as a progressive generalization of iso-superconductivity theory[4,5], by starting with the representation of the intrinsic characteristic of the k-space structure of the deformed Fermi surface as a *deformation* of a Minkowskian metric,

$$diag(1,1,1,-1) \rightarrow diag.(b^2, b^2, b^2, -b_4^2) \qquad (3.1)$$

from which the total energy of a Cooper pair is represented in the form

$$\hat{E}_T = 2mc_o^2 b_4^2 + 2\varepsilon_F / b^2 - E_0 \equiv 2mc_0^2 b_4^2 + \hat{E}_B, \qquad (3.2)$$

where $\hat{E}_B$ is the binding energy of the pair in this isometric space, c_0 being speed of light in vacuum and ε_F the Fermi energy of the electron in vacuum $(b = b_4 = 1)$. The progressive generalization of Eq.(3.2) that takes into account geno-Bragg's law of diffraction for quasicrystals can now be realized as follows:

$$\eta \equiv diag\{1,1,1,-1\} \rightarrow \hat{\eta} \equiv diag\{b_1^2, b_2^2, b_3^2, b_4^2\} \times diag\{1,1,1,-1\} \equiv \hat{I} \times \eta = diag\{16,10,7,-1)$$

which may be rewritten, for iso-superconductivity, in the explicit form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} b_1^2 & 0 & 0 & 0 \\ 0 & b_2^2 & 0 & 0 \\ 0 & 0 & b_3^2 & 0 \\ 0 & 0 & 0 & -b_4^2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (3.3a).$$

and generalized to the "genounit" $(\hat{I}^{>})$, for geno-superconductivity, as follows:

$$\hat{I} \equiv \begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \hat{I}^{>} \equiv \begin{bmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{bmatrix}, \; {}^{<}\hat{I} = (\hat{I}^{>})^{+} \equiv \begin{bmatrix} 16 & 5 & 9 & 4 \\ 3 & 10 & 6 & 15 \\ 2 & 11 & 7 & 14 \\ 13 & 8 & 12 & 1 \end{bmatrix} \quad (3.3b)$$

In the absence of any theoretical prescription for determining the $b_i^2 (i = 1,2,3,4)$ we have correlated them with the diagonal entries in Durer's 4x4 magic square matrix of numbers 1,2,…,16 having the characteristic numbers of various observed splitting/diffraction patterns summarized in Fig.6 & 7. We note from the experimental data in Fig.6 that $Trace(\hat{I}) \equiv b_1^2 + b_2^2 + b_3^2 + b_4^2 = 16 + 10 + 7 + 1 = 34$ = $\frac{1}{2} n(n^2 + 1)$ (for n=4) defines the "iso-unit"($\hat{I}$) in geometrical terms as the number of points, lines and planes defining an inverted glass pyramid representing a deformation of conventional Minkowski space metric $(\eta \rightarrow \hat{\eta} = \hat{I} \times \eta)$, as indicated in Fig. 7.

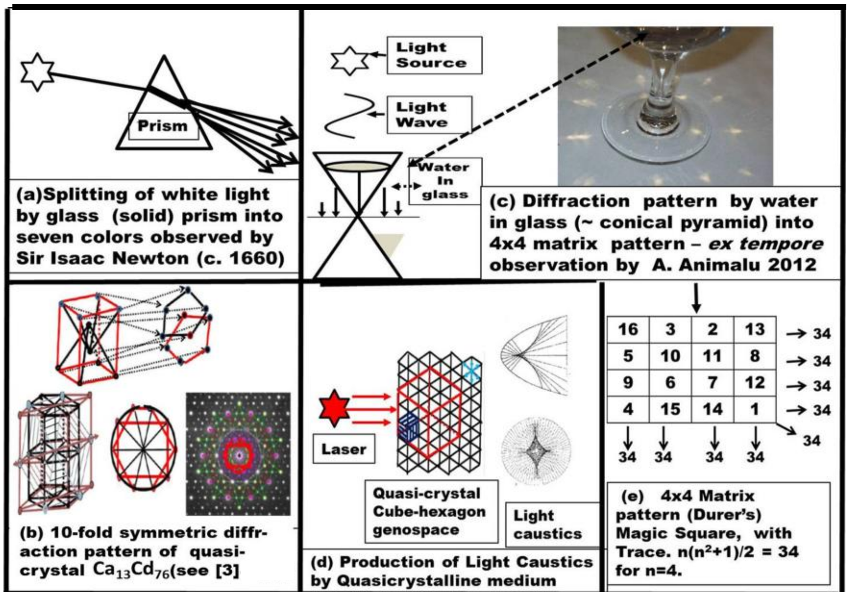


Fig. 6: Experimentally observed splitting and diffraction patterns of light by (a) solid glass prism into Newton's 7 colours of the rainbow, (b)quasi-crystal into 10-fold point symmetry (see, ref.[3] & [10]) (c) conical glass of water into 4x4 = 16 spots[32] , and (d) cube-hexagon quasicrystal into light "caustics"[25]. (e) Correlation with Durer's 4x4 magic square matrix of numbers, 1,2,..,16 characterizing the "iso-unit" and "geno-unit" in Eq.(3.3a &b).

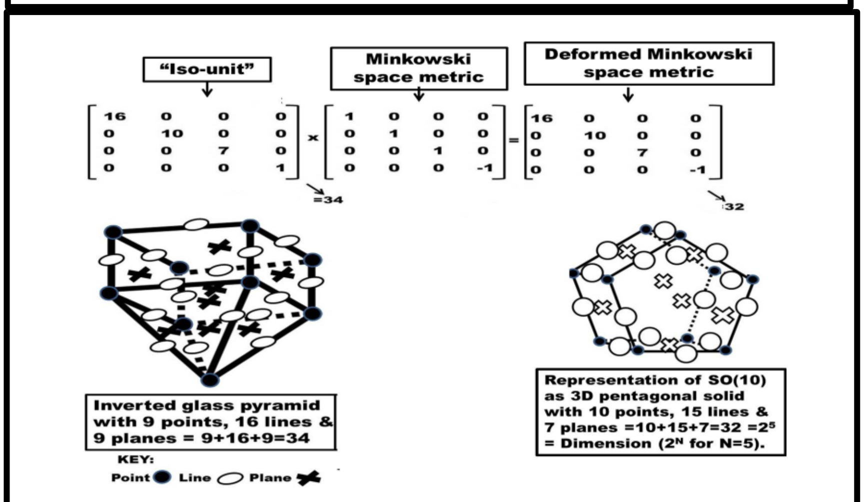
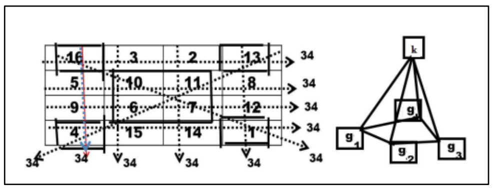


Fig. 7: Identification of Durer's 4x4 magic square matrix of numbers, 1,2,..,16 with the "isounit" and the deformed Minkowski space with SO(10) in terms of the number of the primitive geometric elements (points, lines & planes) of an inverted glass pyramid & pentagonal solid in three-dimensional projective space.

As also indicated in Fig.7, $Trace(\hat{\eta}) \equiv b_1^2 + b_2^2 + b_3^2 - b_4^2 = 16+10+7-1 = 32 = 2^5$, is the dimension of the 3D representation of SO(10) in terms of the number of points, lines and planes defining a pentagonal solid. The fact that such a representation of the progressive generalization of iso- to geno-supercodnuctivity with Durer's magic square is compatible with Bragg's law in the form given by Eq.(1.1), $\vec{g}_i^2 = 2\vec{k}.\vec{g}_i$ is verified by putting $\vec{k} = (\pi\sqrt{34}/\lambda)(1,1,1,1)$ and defining a set of $\vec{g}_i = (2\pi/\lambda\sqrt{34})\vec{p}_i, (i = 1,2,...,12)$ where:

$$\begin{aligned} &\vec{p}_1 = (16,3,2,13), \vec{p}_2 = (5,10,11,8), \vec{p}_3 = (9,6,7,12), \vec{p}_4 = (4,15,14,1), \\ &\vec{p}_5 = (16,5,9,4), \vec{p}_6 = (3,10,6,15), \vec{p}_7 = (2,11,7,14), \vec{p}_8 = (13,8,12,1), \quad (3.4) \\ &\vec{p}_9 = (16,10,7,1), \vec{p}_{10} = (13,11,6,4), \vec{p}_{11} = (16,4,1,13), \vec{p}_{12} = (10,11,7,6), \end{aligned}$$

correspond to the four row vectors and four column vectors as well as the two diagonals vectors and two other 4-vectors of $\hat{I}^{>}$ as shown in the diagram below.



4. GENO-LONDON EQUATION

It is well known from London's(1961)[16] classic Dover-published book entitled *Superfluids, Microscopic Theory of Superconductivity* p. 64, that the diamagnetic current that leads to the Meissner effect in a superconductor ,

$$\vec{J} = (2e\hbar/mc)\vec{A}, \quad (4.1)$$

can be expressed in the (special) relativistic form

$$\frac{\partial p_\nu}{\partial x_\mu} - \frac{\partial p_\mu}{\partial x_\nu} = 0, \;\; p_k = mu_k + \frac{e}{c}A_k, \;\; \frac{e}{i}p_0 = \frac{mc^2}{\sqrt{1-v^2/c^2}} + eA_0 \quad (4.2)$$

where $p_k = mu_k + \frac{e}{c}A_k$ consists of an "inertia" part (mu_k) and a "gauge" part $\frac{e}{c}A_k$.

This equation follows from Maxwell's equation and a correspondence principle

$$(\vec{E},\vec{B}) \leftrightarrow (\vec{\nabla},\vec{p}), i.e., \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix} \leftrightarrow \begin{pmatrix} 0 & \partial_1 & \partial_2 & \partial_3 \\ -\partial_1 & 0 & p_3 & -p_2 \\ -\partial_2 & -p_3 & 0 & p_1 \\ -\partial_3 & p_2 & -p_1 & 0 \end{pmatrix}. \quad (4.3)$$

To generalize the underlying algebraic structure of Eq.(4.2), we observe that Eq.(4.2) can be rewritten in the 2x2 matrix form:

$$0 = (\partial_\mu \quad \partial_\nu)\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} p_\mu \\ p_\nu \end{pmatrix} \equiv \partial_\nu p_\mu - \partial_\mu p_\nu. \quad (4.4)$$

which may be elaborated and transformed as follows:

$$(\partial_\mu p_\mu + \partial_\nu p_\nu) + (\partial_\nu p_\mu - \partial_\mu p_\nu) \equiv \begin{pmatrix} \partial_\mu & \partial_\nu \end{pmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} p_\mu \\ p_\nu \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} \partial_\mu & \partial_\nu \end{pmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} p_\mu \\ p_\nu \end{pmatrix} \equiv \partial_\nu p_\nu - \partial_\mu p_\mu + (\partial_\mu p_\nu + \partial_\nu p_\mu). \quad (4.5)$$

This can be characterized as (*right or left genotopy)*transformation of η:

$$TC\eta \equiv \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \equiv \eta \equiv \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \equiv \eta DT \qquad (4.6a)$$

where $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \equiv \sigma_1; T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \equiv -i\sigma_2; D = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \equiv -\sigma_3.$

are Pauli spin matrices. As a result, we again obtain a Lie-admissible relation.

$$\eta DT - TC\eta = 0, (C \neq D \neq I). \qquad (4.6b)$$

In order to generalize Eq.(4.1) to three dimensions we consider a system characterized by a six-vector (**J**,**B**) which can be represented, in place of the correspondence in Eq.(4.3) by the correspondence principle in Eq.(1.1) in terms of 4x4 antisymmetric tensor analogous to the geno-Einstein field tensor incorporating a dilatation current (D_μ) and conformal current $(K_{\mu\nu})$ given by

$$D_\mu = x^\nu \Theta_{\mu\nu}; K_{\mu\nu} = (2x_\mu x_\nu - g_{\mu\lambda} x^2)\Theta_\nu^\lambda \qquad (4.7a)$$

as follows:

$$\|G_{\mu\nu}\| \equiv \begin{pmatrix} 0 & \Theta_{01} & \Theta_{02} & \Theta_{03} \\ -\Theta_{01} & 0 & D_3 & -D_2 \\ -\Theta_{02} & -D_3 & 0 & D_1 \\ -\Theta_{03} & D_2 & -D_1 & 0 \end{pmatrix} \leftrightarrow \begin{pmatrix} 0 & J_1 & J_2 & J_3 \\ -J_1 & 0 & B_3 & -B_2 \\ -J_2 & -B_3 & 0 & B_1 \\ -J_3 & B_2 & -B_1 & 0 \end{pmatrix} \quad (4.7b)$$

We now recall that the usual Einstein's general relativity field equation for the gravitational field, has the form

$$G_{\mu\nu} \equiv R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = -\kappa \Theta_{\mu\nu} \qquad (4.8a)$$

where $R_{\mu\nu\rho\sigma}$ is the curvature tensor defined by a metric tensor $g_{\mu\nu}$; and $R_{\mu\nu} = g^{\rho\sigma} R_{\rho\sigma\mu\nu}$ is the Ricci tensor and $\kappa = 8\pi G / c^4$ while $\Theta_{\mu\nu}$ is the symmetric energy momentum tensor whose conservation is given by

$$\partial^\nu \Theta_{\mu\nu} = 0 \Rightarrow G_{\mu\nu,\nu} = 0. \qquad (4.8b)$$

Consequently, for scale and conformal invariant field,

$$\partial^{\mu} D_{\mu} = \Theta_{\mu\mu} = 0; \partial^{\nu} K_{\mu\nu} = x^{\beta}\Theta_{\mu\mu} = 0 \qquad (4.8c)$$

Thus, by using the secular determinant equation

$$\mathrm{Det}\left\| G_{\mu\nu} - \lambda\eta_{\mu\nu} \right\| \equiv \lambda^4 - (R_{\mu\nu\rho\sigma} G^{\mu\rho} G^{\nu\sigma})\lambda^2 + (\in_{\mu\nu\rho\sigma} G^{\mu\rho} G^{\nu\sigma})^2 = 0, \qquad (4.9)$$

where $\|\eta_{\mu\nu}\| \equiv diag(+1,-1,-1,-1), R_{\mu\nu\rho\sigma} \equiv (\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\sigma\nu}),$

the analog of geno-Bragg's law(see Appendix A of ref.[10]) becomes

$$R_{\mu\nu\rho\sigma} G^{\mu\rho} G^{\nu\sigma} - 2 \in_{\mu\nu\rho\sigma} G^{\mu\rho} G^{\nu\sigma} = \begin{cases} \mathbf{J}^2 \\ \mathbf{B}^2 \end{cases} \Rightarrow \begin{cases} \mathbf{J}^2 - \mathbf{B}^2 \pm 2\mathbf{B.J} = 0 \\ \mathbf{B}^2 - \mathbf{J}^2 \pm 2\mathbf{J.B} = 0 \end{cases} \qquad (4.10)$$

under the dual transformation ($\mathbf{J} \to \mathbf{B}, \mathbf{B} \to -\mathbf{J}$). For $\mathbf{B.J} \neq 0$, Eq.(4.10) is the geno-London equation that we are after: it relates the supercurrent **J** and **B** in the presence of an electromagnetic vector potential, **A,** as follows:

$$\mathbf{J} = -(2e\hbar / mc)\mathbf{A}, \quad \mathbf{B} = \mathrm{curl}\mathbf{A} \equiv \mathrm{curl}(\zeta\mathbf{J}), \quad \zeta = (mc/2|e|\hbar) \quad (4.11)$$

The well-known fractional quantum Hall effect would be predicted by replacing the electric current **J** carried by an electron in (4.11) by a (magnetic) current (loop) $\mathbf{J^m}$ representing a d-quark by virtue of Eq.(2.20). A practical device based on such a geno-London equation is then obtained by associating such $\mathbf{J^m}$ with a magnetic field **B** according to the *emagnetodynamics* principle[19] of operation of a system of composite magnetic poles consisting of a geno-dual circular array of five magnetic poles and a pivoted rotor invented by Izuogu[20] as elaborated in Fig.8(i). The Izuogu machine has an unexpected feature as a perpetually running pivoted rotor represented in Fig.8(ii) by Kepler's 6-star vortex[32] in the static magnetic field of the geno-dual composite system of magnetic poles.

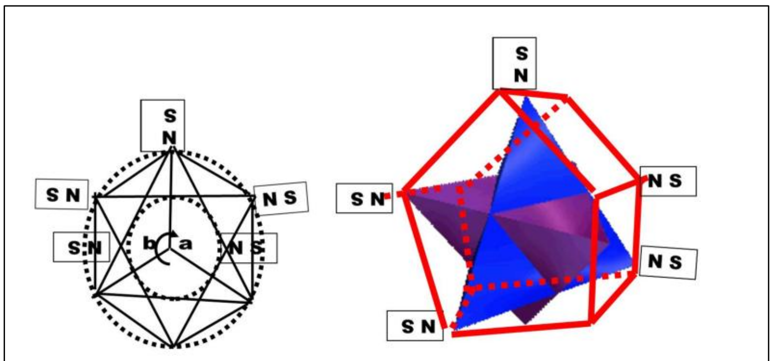


Fig. 8; (i) Izuogu's geno-dual circular array of magnetic poles & pivoted slab of soft iron e-magnetodynamics[20] (ii) Animalu's[19] 3-dimensional characterization of the array of magnetic poles with pivoted slab replaced by Kepler's 6-star vortex[32].

5. DISCUSSION AND CONCLUSION

We have shown in Sec.2 of this paper that the standard BCS model of superconductivity can be generalized straight forwardly to a geno-BCS model, whose novelty lies in its essential time-irreversibility and representation of pairing by current loops. In Sec. 3, we also showed that iso-superconductivity[4,5] can be generalized again straight forwardly to geno-superconductivity, the main interest being the correlation of the iso-unit and geno-unit with the observed diffraction patterns of light by quasi-crystals. In particular, the 4x4=16 diffraction pattern of water in a conical glass enables geno-superconductivity to be realized in a generalization of the usual electromagnetic vector and axial-vector current densities of Dirac's relativistic electron, $\psi^+ J^V_{0\mu}\psi \equiv \psi^+\gamma_0\gamma_\mu\psi$ and $\psi^+ J^A_{0\mu}\psi \equiv \psi^+\gamma_5\gamma_0\gamma_\mu\psi$, as row vector of the corresponding "tensor" current given by the 16x16-matrices:

$$J^V = \begin{bmatrix} \gamma_0^2 & \gamma_0\gamma_1 & \gamma_0\gamma_2 & \gamma_0\gamma_3 \\ \gamma_1\gamma_0 & \gamma_1^2 & \gamma_1\gamma_2 & \gamma_1\gamma_3 \\ \gamma_2\gamma_0 & \gamma_2\gamma_1 & \gamma_2^2 & \gamma_2\gamma_3 \\ \gamma_3\gamma_0 & \gamma_3\gamma_1 & \gamma_3\gamma_2 & \gamma_3^2 \end{bmatrix}, \quad J^A = \begin{bmatrix} \gamma_5\gamma_0^2 & \gamma_5\gamma_0\gamma_1 & \gamma_5\gamma_0\gamma_2 & \gamma_5\gamma_0\gamma_3 \\ \gamma_5\gamma_1\gamma_0 & \gamma_5\gamma_1^2 & \gamma_5\gamma_1\gamma_2 & \gamma_5\gamma_1\gamma_3 \\ \gamma_5\gamma_2\gamma_0 & \gamma_5\gamma_2\gamma_1 & \gamma_5\gamma_2^2 & \gamma_5\gamma_2\gamma_3 \\ \gamma_5\gamma_3\gamma_0 & \gamma_5\gamma_3\gamma_1 & \gamma_5\gamma_3\gamma_2 & \gamma_5\gamma_3^2 \end{bmatrix}, \tag{4.12}$$

As is well-known from conventional Vector-Axial (V-A) Vector current theory of weak decay, the Dirac vector current is conserved while the axial vector current is only partially conserved; and for tunneling of Cooper pairs across a Josephson junction, the axial vector current is formally identifiable(Animalu(1973)[33]) with the phase-dependent Josephson current and bias voltage as given by Anderson(1970)[34]:

$$I = I_0 \sin\varphi, \quad V = (\hbar/2e)d\varphi/dt. \tag{4.13}$$

We are thus led to the conclusion that geno-superconductivity theory has a sufficiently deep foundation to be incorporated in the current search for room temperature superconductor, with water at room temperature in a conical glass (~inverted pyramid) in a gravitational field as a specific example[35].

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Can doping graphite trigger room temperature superconductivity?
Evidence for granular high-temperature superconductivity in water-treated graphite powder.